

# General bounds for sender-receiver capacities in multipoint quantum communications

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We investigate the maximum rates for transmitting quantum information, distilling entanglement and distributing secret keys between a sender and a receiver in a multipoint communication scenario, with the assistance of unlimited two-way classical communication involving all parties. First we consider the case where a sender communicates with an arbitrary number of receivers, so called quantum broadcast channel. Here we also provide a simple analysis in the bosonic setting where we consider quantum broadcasting through a sequence of beamsplitters. Then, we consider the opposite case where an arbitrary number of senders communicate with a single receiver, so called quantum multiple-access channel. Finally, we study the general case of a quantum interference channel where an arbitrary number of senders communicate with an arbitrary number of receivers. Since our bounds are formulated for quantum systems of arbitrary dimension, they can be applied to many different physical scenarios involving multipoint quantum communication.

Today a huge effort is devoted to the development of robust quantum technologies, inspired by quantum information theory [1–5]. The most typical tasks are quantum key distribution (QKD) [6–16], reliable transmission of quantum information [17, 18] and distillation of entanglement [19–21], by means of which two parties are able to implement quantum teleportation [22–24]. Unfortunately, any quantum information task is unavoidably affected by decoherence [25]. For this reason, in any point-to-point quantum communication scenario, i.e., in the absence of quantum repeaters [26], the rates at which entanglement and secret keys can be distributed suffer from limitations, especially when the distance is increased.

In this regard, it is therefore a crucial problem to find the optimal rates for entanglement and key distribution which are achievable by two remote parties, say Alice and Bob, assuming that they are generally assisted by unlimited two-way classical communication (CCs). This investigation started with the determination of achievable lower bounds based on the coherent information [27] and the reverse coherent information [28, 29], continuing with the design of upper bounds based on the squashed entanglement [30, 31]. Finally, Ref. [32] solved the problem and established the maximum rates (two-way capacities) at which two parties can generate keys ( $K$ ), distill entanglement ( $D_2$ ) and transmit quantum information ( $Q_2$ ) over bosonic lossy channels with arbitrary transmissivity  $\eta$ : These are all equal to  $-\log_2(1 - \eta)$ , corresponding to  $\simeq 1.44\eta$  bits per channel use at high loss. This fundamental rate-loss scaling affects QKD and any other quantum optical communication in the absence of repeaters.

To investigate the various two-way capacities, Ref. [32] introduced a novel reduction method whose application can be adapted to many other scenarios. The first ingredient is the definition of the relative entropy of entanglement (REE) of a quantum channel, by extending the notion for quantum states [33–35]. The channel’s REE provides a general upper bound for all the two-way capacities of an arbitrary quantum channel. The second ingredient is a technique of “teleportation stretching” which applies to quantum channels which suitably com-

mute with teleportation. By extending and generalizing previous ideas [20], teleportation stretching allows one to reduce any quantum protocol based on local operations (LOs) and two-way CCs, briefly called adaptive LOCCs, into a much simpler block protocol, where all the transmissions through the channel are transformed into Choi matrices [36] and the adaptive LOCCs are collapsed into a single and final LOCC. Ref. [32] showed this technique for an adaptive protocol with a completely arbitrary task and which is implemented over a wide family of quantum channels in arbitrary dimension, finite or infinite.

Combining the previous two ingredients together, the resulting “REE+teleportation” method allows one to reduce the channel’s REE to the computation of a simple one-shot quantity. This is the REE of the Choi matrix of the channel, dubbed entanglement flux [32]. This approach led to the computation of the two-way capacities ( $K$ ,  $D_2$ ,  $Q_2$ ) of the lossy channel, the quantum-limited amplifier, and the dephasing channel in arbitrary dimension, as well as the secret key capacity  $K$  of the erasure channel in arbitrary dimension [32, 37] (for the specific case of the erasure channel see also the tight upper bound independently derived in Ref. [31]). The “REE+teleportation” method also allowed to write the tightest upper bounds for depolarizing channels in arbitrary dimension, qubit Pauli channels and all one-mode Gaussian channels [32, 37].

In this paper we extend this method to more complex communication scenarios, where multiple senders and/or receivers are involved in the process. The basic configurations are represented by the quantum broadcast channel [38–41] where information is broadcast from a single sender to multiple receivers and the quantum multiple-access channel [42, 43] through which multiple senders communicate with a single receiver. More generally, we also consider the combination of these two cases, where many senders communicate with many receivers in a sort of all-in-all quantum communication, known as quantum interference channel [44–47]. In practical implementations, this may represent a quantum bus [48, 49] where quantum information is transmitted among an ar-

bitrary number of qubit registers.

In all these multipoint communication scenarios, we characterize the most general quantum protocol for key distribution, entanglement distillation and quantum communication, which is assisted by adaptive LOCCs. This leads to the definition of the two-way capacity  $\mathcal{C} = K, D_2$ , or  $Q_2$  between any pair of sender and receiver. We then consider those quantum channels (for broadcasting, multiple-accessing, and all-in-all communication) which suitably commute with teleportation. These are called “stretchable” following the notation of Ref. [32]. For these channels, we can extend the technique of teleportation stretching and completely reduce any adaptive protocol into a block form which involves tensor product of generalized Choi matrices. Combining this reduction with the REE, we can then extend the method of Ref. [32] and bound all the two-way capacities of these channels with the REE of their Choi matrix or entanglement flux [32, 37].

It is important to note that our upper bounds are completely general and can be applied to both discrete-variable (DV) and continuous-variable (CV) systems. As a specific example, we also consider the specific case of a 1-to- $M$  lossy broadcast channel thorough a sequence of beamsplitters or “multisplitter”. In particular, we discuss how that the two-way capacities  $Q_2, D_2$  and  $K$  between the sender and each receiver are all bounded by the entanglement flux of the first point-to-point lossy channel in the multisplitter. This means that, as expected, the fundamental rate-loss scaling of  $-\log_2(1-\eta)$  also applies to lossy broadcast channels.

The paper is organized as follows. We consider the quantum broadcast channel in Sec. I, the quantum multiple-access channel in Sec. II and, finally, the quantum interference channel in Sec. III. Finally, Sec. IV is for conclusions and discussions.

## I. QUANTUM BROADCAST CHANNEL

We can easily adapt the technique developed in [32, 37] to bound the optimal rates in the case of multiple receivers, i.e., for a point-to-multipoint quantum protocol. In fact, consider a quantum broadcast channel  $\mathcal{E}$  where Alice (system ensemble  $\mathbf{a}$ ) transmits a system  $a \in \mathbf{a}$  to  $M$  different Bobs; the generic  $i$ th Bob (with  $i = 1, \dots, M$ ) receives an output system  $b^i$  which may be combined with a local ensemble  $\mathbf{b}^i$  for coherent manipulation. Denote by  $\mathcal{D}(\mathcal{H}_s)$  the space of density operators defined over the Hilbert space  $\mathcal{H}_s$  of system  $s$ . Then, the quantum broadcast channel is a completely-positive trace preserving (CPTP) map from Alice’s input space  $\mathcal{D}(\mathcal{H}_a)$  to the Bobs’ output space  $\mathcal{D}(\otimes_i \mathcal{H}_{b^i})$ . The most general adaptive protocol over this channel goes as follows.

All the parties prepare their initial systems by means of a LOCC  $\Lambda_0$ . Then, Alice picks the first system  $a_1 \in \mathbf{a}$  which is broadcast to all Bobs  $a_1 \rightarrow \{b_1^i\}$  through channel  $\mathcal{E}$ . This is followed by another LOCC  $\Lambda_1$  involving

all parties. Bobs’ ensembles are updated as  $b_1^i \mathbf{b}^i \rightarrow \mathbf{b}^i$ . Then, there is the second broadcast  $\mathbf{a} \ni a_2 \rightarrow \{b_2^i\}$  through  $\mathcal{E}$ , followed by another LOCC  $\Lambda_2$  and so on. After  $n$  uses, Alice and the  $i$ th Bob share an output state  $\rho_{\mathbf{ab}^i}^n$  which is epsilon-close to a target state with  $nR_i^n$  bits. The generic broadcast capacity for the  $i$ th Bob is defined by maximizing the asymptotic rate over all the adaptive LOCCs  $\mathcal{L} = \{\Lambda_0, \Lambda_1, \dots\}$ , i.e., we have

$$\mathcal{C}^i := \sup_{\mathcal{L}} \lim_n R_i^n. \quad (1)$$

In particular, by specifying the adaptive protocol to a particular task, one derives the entanglement-distillation broadcast capacity ( $D_2^i$ ), the quantum broadcast capacity ( $Q_2^i$ ) and the secret-key broadcast capacity ( $K^i$ ). These are all assisted by unlimited two-way CCs between the parties and must satisfy

$$D_2^i = Q_2^i \leq K^i. \quad (2)$$

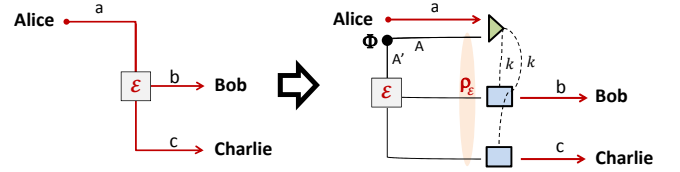


FIG. 1: Stretchable quantum broadcast channel. We can replace the broadcast  $a \rightarrow bc$  over  $\mathcal{E}$  by teleportation over its Choi matrix  $\rho_{\mathcal{E}}$  with CCs to Bob and Charlie. These will implement generally-different correction unitaries.

To bound the previous capacities, let us introduce the notion of stretchable broadcast channel. It is explained for the case of two receivers, Bob and Charlie, with the extension to arbitrary  $M$  receivers being just a matter of technicalities. This is a broadcast channel which commutes with teleportation. This means that, for any teleportation unitary  $U_k$ , we may write

$$\mathcal{E}(U_k \rho U_k^\dagger) = (B_k \otimes C_k) \mathcal{E}(\rho) (B_k \otimes C_k)^\dagger, \quad (3)$$

for unitary  $B_k$  and  $C_k$ . If this is the case, then we can replace  $\mathcal{E}$  with teleportation over its Choi matrix

$$\rho_{\mathcal{E}} = (I_A \otimes \mathcal{E}_{A'}) (\Phi_{AA'}), \quad (4)$$

which is defined by sending half of an EPR  $\Phi_{AA'}$  through the broadcast channel, as also depicted in Fig. 1.

We can easily simplify the previous adaptive protocol when performed over a stretchable broadcast channel. In fact, we can repeat the derivation of Refs. [32, 37] and derive the stretching depicted in Fig. 2. Thus the total output state of Alice, Bob and Charlie reads

$$\rho_{\mathbf{abc}}^n := \rho_{\mathbf{abc}} (\mathcal{E}^{\otimes n}) = \bar{\Lambda} (\rho_{\mathcal{E}}^{\otimes n}), \quad (5)$$

where  $\bar{\Lambda}$  is a trace-preserving LOCC. If we now trace one of the two receivers, e.g., Charlie, we still have a trace-preserving LOCC  $\bar{\Lambda}_{\mathbf{ab}}$  between Alice and Bob, and we may write the following

$$\rho_{\mathbf{ab}}^n = \bar{\Lambda}_{\mathbf{ab}} (\rho_{\mathcal{E}}^{\otimes n}). \quad (6)$$

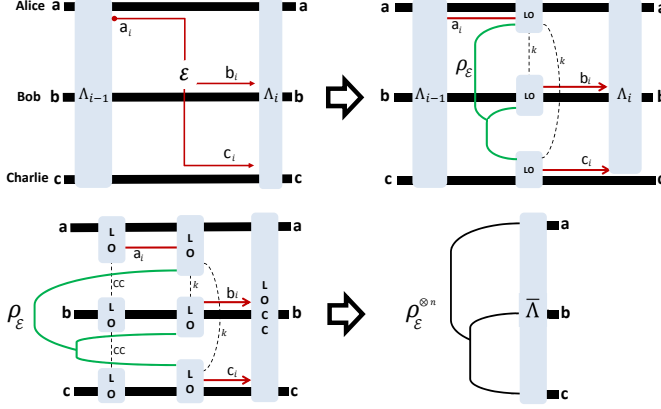


FIG. 2: Stretching an adaptive protocol over a quantum broadcast channel. **Top panels.** The generic  $i$ th transmission  $a_i \rightarrow \{b_i, c_i\}$  over the stretchable broadcast channel  $\mathcal{E}$  (red line) is replaced by a teleportation over its Choi matrix  $\rho_{\mathcal{E}}$  (following the procedure shown in Fig. 1). The input system and the upper half of the Choi matrix are subject to a Bell detection which becomes part of Alice's LO (upper LO). The result of the Bell detection  $k$  is classically communicated to Bob and Charlie so that they can apply two correction unitaries which are then included into their respective LOs (middle and lower LOs). **Bottom panels.** The Choi matrix is stretched in time out of the adaptive LOCCs which are then collapsed into a single trace-preserving LOCC. After  $n$  uses, we can express the output in terms of  $n$  copies of the Choi matrix  $\rho_{\mathcal{E}}$  of the broadcast channel, plus a trace-preserving single final LOCC  $\bar{\Lambda}$  as in Eq. (5).

For Alice and Bob ( $i = B$ ), we can now exploit the weak converse Theorem proven in Ref. [32] which bounds the capacity by means of the REE  $E_R$  and leads to the following

$$C^B \leq \sup_{\mathcal{L}} \limsup_{n \rightarrow +\infty} \frac{E_R(\rho_{\mathbf{ab}}^n)}{n} \leq E_R(\rho_{\mathcal{E}}) := \Phi(\mathcal{E}), \quad (7)$$

where  $\Phi(\mathcal{E})$  is the entanglement flux of the broadcast channel, defined as the REE of its Choi matrix. Note that we find the same bound for the capacity of Alice and Charlie ( $i = C$ ). It is clear that the derivation can be extended to arbitrary  $M$  receivers, for which we write

$$C^i \leq \Phi(\mathcal{E}). \quad (8)$$

### A. Broadcasting in a bosonic lossy environment

Let us consider a 1-to- $M$  lossy bosonic broadcast channel from Alice to an arbitrary number  $M$  of Bobs. One possible physical representation is a chain of  $M+1$  beam splitters with transmissivities  $(\eta_0, \eta_1, \dots, \eta_M)$  in which Alice's input mode subsequently interacts with  $M+1$  vacuum modes. The  $M$  output modes ( $B_1, B_2, \dots, B_M$ ) are then given to the different Bobs, with the extra modes  $E$  and  $E'$  being the leakage to the environment (or an

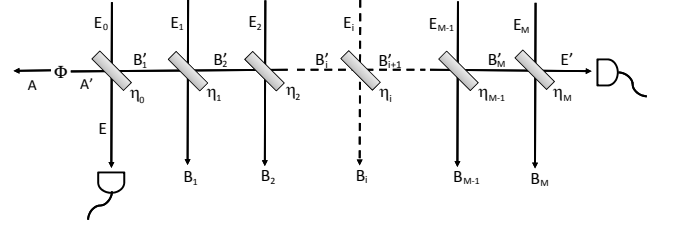


FIG. 3: Lossy bosonic broadcast channel from one sender (Alice, mode  $A'$ ) to  $M$  receivers (Bobs, modes  $B_1, \dots, B_M$ ), modelled as a multi-splitter, i.e., a sequence of  $M+1$  beamsplitters with transmissivities  $(\eta_0, \eta_1, \dots, \eta_M)$ . The environmental input modes  $E_0, E_1, \dots, E_M$  are all in the vacuum state. Modes  $E$  and  $E'$  describe leakage to the environment.

eavesdropper). See Fig. 3 for a schematic.

Let us denote the  $M$ -broadcast channel with the notation  $\mathcal{E}_{A' \rightarrow B_1 \dots B_M}$ . Its entanglement flux is given by

$$\Phi(\mathcal{E}_{A' \rightarrow B_1 \dots B_M}) = E_R(\rho_{AB_1 \dots B_M}), \quad (9)$$

where

$$\rho_{AB_1 \dots B_M} = (I_A \otimes \mathcal{E}_{A' \rightarrow B_1 \dots B_M})(\Phi_{AA'}) \quad (10)$$

is the Choi matrix of the  $M$ -receiver broadcast channel defined as in Eq. (4). We now show that the quantity on the r.h.s. of the Eq. (8) can be upper bounded by the entanglement flux  $\Phi(\mathcal{E}_{A' \rightarrow B'_1})$  of the first lossy channel  $\mathcal{E}_{A' \rightarrow B'_1}$  with transmissivity  $\eta_0$ . As we know from Ref. [32], this coincides with all the two-way capacities of this channel, and we have

$$\Phi(\mathcal{E}_{A' \rightarrow B'_1}) = \mathcal{C}(\mathcal{E}_{A' \rightarrow B'_1}) = -\log(1 - \eta_0). \quad (11)$$

To show this we first exploit the fact that the Choi matrix  $\rho_{AB_1 \dots B_i B'_{i+1}}$  associated to the broadcast channel from sender  $A$  to the receivers  $B_1 \dots B_i B'_{i+1}$  can be seen as the result of a local operation  $\mathbf{L}_{\{A \dots B_{i-1} B'_i\} E_i}$ , at the  $i$ th beam splitter, between the input states of modes  $A \dots B_{i-1} B'_i$  and the  $i$ th vacuum mode  $E_i$ . Thus, at the  $M$ th beam splitter we can write

$$\rho_{AB_1 \dots B_M} = \mathbf{L}_{\{A \dots B_{M-1} B'_M\} E_M} (\rho_{AB_1 \dots B_{M-1} B'_M} \otimes |0\rangle\langle 0|_{E_M}). \quad (12)$$

The same holds for  $\rho_{AB_1 \dots B_{M-1} B'_M}$  at the  $(M-1)$ th beam splitter and so on, which allows us to write

$$\begin{aligned} \rho_{AB_1 \dots B_{M-1} B'_M} &= \mathbf{L}_{\{A \dots B_{M-2} B'_{M-1}\} E_{M-1}} (\rho_{A \dots B_{M-2} B'_{M-1}} \otimes |0\rangle\langle 0|_{E_{M-1}}) \\ &\vdots \\ \rho_{AB_1 B'_2} &= \mathbf{L}_{\{A B'_1\} E_1} (\rho_{A B'_1} \otimes |0\rangle\langle 0|_{E_1}). \end{aligned} \quad (13)$$

Then it's clear how the output state  $\rho_{AB_1 \dots B_M}$  is obtained from  $\rho_{A B'_1}$  by means of a composition of subsequent LOs resulting in a global LO  $\Xi^M$ , and in a more

compact form

$$\rho_{AB_1 \dots B_M} = \Xi^M[\rho_{AB'_1}], \quad (14)$$

where

$$\Xi^M[\rho_{AB'_1}] = \tilde{\mathbf{L}}^M \circ \tilde{\mathbf{L}}^{M-1} \circ \dots \circ \tilde{\mathbf{L}}^1[\rho_{AB'_1}], \quad (15)$$

and

$$\tilde{\mathbf{L}}^i[\cdot] = \mathbf{L}_{\{A \dots B_{i-2} B'_{i-1}\} B_{i-1} B'_i}(\cdot \otimes |0\rangle\langle 0|_{E_i}). \quad (16)$$

Now by exploiting the monotonicity of the REE under trace-preserving LOs, we may write

$$E_R(\rho_{AB_1 \dots B_M}) \leq E_R(\rho_{AB'_1}). \quad (17)$$

Then, recalling Eq. (11), we have that the generic capacity between Alice and the  $i$ th Bob is upper bounded as follows

$$\mathcal{C}^i \leq \Phi(\mathcal{E}_{A' \rightarrow B_1 \dots B_M}) \leq \Phi(\mathcal{E}_{A' \rightarrow B'_1}) = -\log(1 - \eta_0). \quad (18)$$

As expected, this shows that the first beam splitter is a universal bottleneck which restricts the capacities between Alice and any of the receiving Bobs.

## II. QUANTUM MULTIPLE-ACCESS CHANNEL

We can also study a multipoint-to-point quantum protocol, i.e., a quantum multiple-access channel. Here we have the opposite scenario with respect to the previous quantum broadcast channel: There are  $M$  senders (Alices) and a single receiver (Bob). This channel is a CPTP map from Alices' input space  $\mathcal{D}(\otimes_i \mathcal{H}_{a^i})$  to Bob's output space  $\mathcal{D}(\mathcal{H}_b)$ . The most general adaptive protocol over this channel goes as follows.

All the parties prepare their initial systems by means of a LOCC  $\Lambda_0$ . Then, the  $i$ th Alice picks the first system from her local ensemble, i.e.,  $a^i_1 \in \mathbf{a}^i$ . All Alice's input systems are sent through the quantum multiple-access channel  $\mathcal{E}$  with output  $b_1$  for Bob, i.e.,

$$a^1_1, \dots, a^i_1, \dots, a^M_1 \xrightarrow{\mathcal{E}} b_1. \quad (19)$$

This is followed by another LOCC  $\Lambda_1$  involving all parties. Bob ensemble is updated as  $b_1 \mathbf{b} \rightarrow \mathbf{b}$ . Then, there is the second transmission  $\{\mathbf{a}^i\} \ni \{a^i_2\} \rightarrow b_2$  through  $\mathcal{E}$ , followed by another LOCC  $\Lambda_2$  and so on. After  $n$  uses, the  $i$ th Alice and Bob share an output state  $\rho_{\mathbf{a}^i \mathbf{b}}^n$  which is epsilon-close to a target state with  $nR_i^n$  bits.

The generic multiple-access capacity for the  $i$ th Alice is defined by maximizing the asymptotic rate over all the adaptive LOCCs  $\mathcal{L} = \{\Lambda_0, \Lambda_1, \dots\}$ , i.e., we have

$$\mathcal{C}^i := \sup_{\mathcal{L}} \lim_n R_i^n. \quad (20)$$

As before, by specifying the adaptive protocol to a particular task, one derives the entanglement distillation

multiple-access capacity ( $D_2^i$ ), the quantum multiple-access capacity ( $Q_2^i$ ) and the secret-key multiple-access capacity ( $K^i$ ). These are assisted by unlimited two-way CCs between the parties and must satisfy the usual relation  $D_2^i = Q_2^i \leq K^i$ .

Let us introduce the notion of stretchable quantum multiple-access channel. For the sake of simplicity, this is explained for the case of two senders, with the extension to arbitrary  $M$  senders being just a matter of technicalities. A quantum multiple-access channel is stretchable if commutes with teleportation, i.e., for any teleportation unitaries  $U_{k_1}^1$  and  $U_{k_2}^2$  we may write

$$\mathcal{E}[(U_{k_1}^1 \otimes U_{k_2}^2) \rho (U_{k_1}^1 \otimes U_{k_2}^2)^\dagger] = V_k \mathcal{E}(\rho) V_k^\dagger, \quad (21)$$

for unitary  $V_k$  with  $k$  being a function of  $k_1$  and  $k_2$ . If this is the case, then we can replace  $\mathcal{E}$  with teleportation over its Choi matrix, which is defined by sending halves of two EPR states through the channel, i.e.,

$$\rho_{\mathcal{E}} = (I_{A^1 A^2} \otimes \mathcal{E}_{A'^1 A'^2})(\Phi_{A^1 A'^1} \otimes \Phi_{A^2 A'^2}). \quad (22)$$

See also Fig. 4 for further explanations.

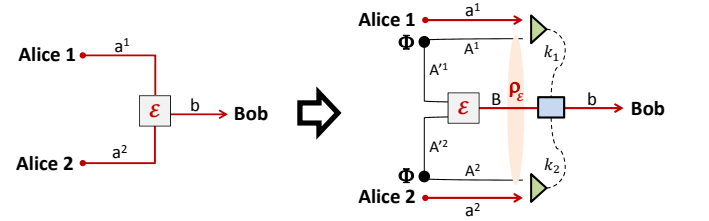


FIG. 4: Stretchable quantum multiple-access channel. We can replace the multiple-access channel  $\mathcal{E} : a^1 a^2 \rightarrow b$  (left) by double teleportation over its tripartite Choi matrix  $\rho_{\mathcal{E}}$  (right). This Choi matrix is obtained by sending halves ( $A'^1$  and  $A'^2$ ) of two EPR states  $\Phi$  through  $\mathcal{E}$ , with output  $B$ . Then, systems  $a^1$  and  $A^1$  are subject to a Bell detection with outcome  $k_1$ . Similarly, systems  $a^2$  and  $A^2$  are subject to a Bell detection with outcome  $k_2$ . The outcomes are CCed to Bob who applies a correction unitary on system  $B$ . Since the channel is stretchable, i.e., it commutes with the teleportation unitaries according to Eq. (21), Bob's correction unitary  $V_k$  on  $B$  re-generates the original channel  $\mathcal{E} : a^1 a^2 \rightarrow b$ .

We can simplify an adaptive protocol performed over a stretchable quantum multiple-access channel  $\mathcal{E}$ . In fact, each transmission through  $\mathcal{E}$  can be replaced by a double teleportation on  $\rho_{\mathcal{E}}$ , with the Bell detections and Bob's correction unitary being included in the adaptive LOCCs. By stretching  $n$  uses (see Fig. 5), we find that the total output state of Alice 1, Alice 2 and Bob reads

$$\rho_{\mathbf{a}^1 \mathbf{a}^2 \mathbf{b}}^n = \bar{\Lambda}(\rho_{\mathcal{E}}^{\otimes n}). \quad (23)$$

If we now trace one of the two senders, e.g., Alice 2, we still have a LOCC  $\bar{\Lambda}_{\mathbf{a}^1 \mathbf{b}}$  between Alice 1 and Bob, and we may write the following

$$\rho_{\mathbf{a}^1 \mathbf{b}}^n = \bar{\Lambda}_{\mathbf{a}^1 \mathbf{b}}(\rho_{\mathcal{E}}^{\otimes n}). \quad (24)$$





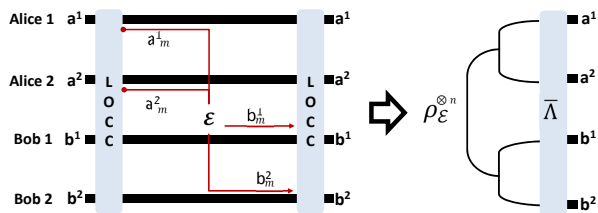


FIG. 7: Stretching an adaptive protocol over a quantum interference channel (generic  $m$ th transmission shown on the left). After  $n$  uses, we can express the output in terms of  $n$  copies of the Choi matrix  $\rho_{\mathcal{E}}$  of the quantum interference channel, subject to a trace-preserving LOCC  $\bar{\Lambda}$ .

It follows that the capacity for Alice 1 and Bob 1 ( $i = j = 1$ ) is upper bounded as

$$\mathcal{C}^{11} \leq \sup_{\mathcal{L}} \limsup_{n \rightarrow +\infty} \frac{E_R(\rho_{\mathbf{a}^1 \mathbf{b}^1}^n)}{n} \leq E_R(\rho_{\mathcal{E}}) := \Phi(\mathcal{E}), \quad (33)$$

where  $\Phi(\mathcal{E})$  is the entanglement flux of the quantum interference channel. Clearly, we find the same result in all other cases, i.e., for any pair  $(i, j)$ .

#### IV. CONCLUSIONS

In this work we have considered the capacities for quantum communication, entanglement distillation and secret key generation between any sender-receiver pair in a general multipoint scenario, where all parties are assisted by unlimited two-way CCs. We have considered the quantum broadcast channel (point-to-multipoint), the multiple-access channel (multipoint-to-point) and the

quantum interference channel (multipoint-to-multipoint) assuming quantum systems of arbitrary dimension, i.e., finite or infinite. By using the “REE+teleportation” method of Ref. [32], which suitably combines the relative entropy of entanglement (REE) with teleportation stretching, we have reduced the most general adaptive protocols implemented on these multipoint channels to the computation of a one-shot quantity, their entanglement flux (defined by the REE of their Choi matrix).

Further research should be directed to show that the “REE+teleportation” reduction method can be used as a general tool to upperbound the entire capacity regions of the previous multipoint channels in arbitrary dimension. These regions are defined as the convex closure of the set of all the rates which are achievable by the parties assisted by unlimited two-way CCs. The result obtained for the lossy broadcast channel [50] is a promising step in this direction. By using the REE+teleportation method for the upper bound [32] and the quantum state merging protocol for the lower bound [51], Ref. [50] established the capacity region of the lossy broadcast channel [52].

This field is still in its infancy. It is an open question to determine the two-way assisted capacity regions for Gaussian broadcast channels, for instance including thermal or additive noise. It is also an open question to determine these regions for broadcast channels in the discrete-variable setting (qubits/qudits). Similarly, all the analyses should also be extended to the quantum multiple-access channel and the quantum interference channel.

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